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CONTENT-ADAPTIVE MESH MODELING FOR FULLY-3D TOMOGRAPHIC IMAGE RECONSTRUCTION

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ABSTRACT

In this paper we propose the use of a content-adaptive volumetric mesh model for fully three-dimensional (3D) tomographic image reconstruction. In the proposed framework, the image to be reconstructed is first modeled by an efficient mesh representation. The image is then obtained through estimation of the nodal values from the measured data. The use of a mesh representation can alleviate the ill-posed nature of the reconstruction problem, thereby leading to improved quality in the reconstructed images. In addition, it reduces the data storage requirement, resulting in efficient algorithms. The proposed methods are tested using gated cardiac-perfusion images. Initial results demonstrate that the proposed approach achieves superior performance when compared to several commonly used methods for image reconstruction, and produces results very rapidly.

1. INTRODUCTION

In recent years there has been growing interest in fully-3D tomographic image reconstruction. A major challenge in fully-3D reconstruction lies in its memory requirement and demanding computation time. Like their 2D counterpart, most 3D reconstruction methods have traditionally been developed based on voxel image representations [1]. Bayesian priors (e.g., [2]) or regularization terms (e.g., [3]) are typically used to combat the effect of noise.

Alternative model-based reconstruction approaches have also been proposed. For example, cylindrical models were proposed in [4] and surface models were used in [5,6].

In our previous work in [7], a content-adaptive mesh modeling approach was proposed for 2D image reconstruction. It was demonstrated that such an approach can outperform several well-known reconstruction algorithms in terms of both reconstructed image quality and computation time. In this study, we extend this approach to fully-3D image reconstruction. In this new approach, the image is first modeled by a volumetric mesh model, on the basis of which a customized basis representation is obtained for the image. The parameters of this representation are then estimated from the data.

In a volumetric mesh model, the 3D image domain is subdivided into a collection of mesh elements, the vertices of which are called nodes. The image function is then obtained over each element by interpolation from the values of these nodes [8]. In a content-adaptive mesh model (CAMM), the mesh elements are placed in a fashion that is adapted to the local content of the image. A CAMM provides an efficient representation of the image in that the number of parameters (i.e., mesh nodes) is typically much less than the number of required voxels in a voxel image representation. In addition, a mesh model can also be used for motion tracking in an image sequence, by allowing the mesh to deform over time [9].

The potential benefits of using a CAMM for image reconstruction are: 1) a CAMM reduces the number of unknowns, thus alleviating both the underdetermined nature of the reconstruction problem and the data storage requirement, particularly for the case of 3D reconstruction; 2) this reduction in the number of unknowns can also lead to a fast computation; 3) a CAMM provides a natural spatially-adaptive smoothness mechanism; and 4) the CAMM provides a natural framework for reconstruction of moving image sequences.

2. METHODS

2.1 Mesh Tomography Model

Let \( f(\mathbf{x}) \) denote the image function defined over a domain \( D \), which is 3D in this study. In a mesh model, the domain \( D \) is partitioned into \( M \) non-overlapping mesh elements,
denoted by $D_m$, $m=1,2,\cdots,M$. The image function is represented as

$$f(x) = \sum_{s=1}^{N} f(x_s) \varphi_s(x) + e(x),$$  \hspace{1cm} (1)$$

where $x_s$ is the $s$th mesh node, $\varphi_s(x)$ is the interpolation basis function associated with $x_s$, $N$ is the total number of mesh nodes used, and $e(x)$ is the modeling error. Note that the support of each basis function $\varphi_s(x)$ is limited to those elements $D_m$ attached to the node $n$. In this study, tetrahedrons are used for $D_m$, and linear interpolation functions are used for $\varphi_s(x)$.

Now let $n$ denote a vector formed by the nodal values of the mesh model, i.e.,

$$n = \left[ f(x_1), f(x_2), \cdots f(x_s) \right]^T.$$  \hspace{1cm} (2)$$

If $f$ denotes the voxel representation of the image function $f(x)$ over $D$, then from (1) and (2) one can obtain

$$f = \Phi n + e,$$  \hspace{1cm} (3)$$

where $\Phi$ is a matrix, composed from the interpolation functions $\varphi_s(x)$ in (1), that forms the interpolation operator from a mesh representation to the pixel representation, and $e$ is a vector denoting the error $e(x)$.

For tomographic image reconstruction, the imaging equation is typically written in terms of the voxel representation $f$ as

$$E[g] = Hf,$$  \hspace{1cm} (4)$$

where $g$ contains the measured data, $E[\cdot]$ is the expectation operator, and $H$ is a matrix describing the imaging system.

Substituting (3) into (4), we obtain the mesh-domain imaging equation:

$$E[g] = H [\Phi n + e] = An + e,$$  \hspace{1cm} (5)$$

where $A = H\Phi$, and $e = He$.

As demonstrated later, a CAMM can provide a very accurate representation of the original image. As a result, the modeling error $e$ in (5) can be ignored when compared to the noise level in the imaging data. Thus, we have

$$E[g] \approx An.$$  \hspace{1cm} (6)$$

The reconstruction problem becomes that of estimating $n$ from the observed data $g$. The image $f$ can then be obtained from (3) (with $e$ ignored).

### 2.2 Reconstruction Algorithms

In this paper we investigate maximum-likelihood and least-squares estimates of the nodal values in $n$.

#### A. Maximum-Likelihood Estimate

The maximum-likelihood (ML) estimate is obtained as

$$\hat{n}_{ml} = \arg \max_n \left\{ \log \left[ \rho(g;n) \right] \right\},$$  \hspace{1cm} (7)$$

where $\rho(g;n)$ is the likelihood function of $g$ parameterized by $n$. In this paper, we assume a Poisson likelihood, which characterizes emission tomography.

The ML estimate can be computed by using the following expectation-maximization (EM) algorithm [10]:

$$n^{(j+1)}_s = \frac{n^{(j)}_s + \sum_{t} A_{nt} \sum_{k} g_k \delta_{nk}^{(j)}}{\sum_{t} A_{nt}^{(j)}},$$  \hspace{1cm} (8)$$

where $n^{(j)}_s$ is the value of node $s$ in iteration $j$, $g_k$ is the recorded count for observation $t$, and $A_{nt}$ is the $ts$ entry of matrix $A$.

#### B. Least-Squares Estimate

The least-squares estimate is obtained as the solution of the following optimization problem:

$$\hat{n}_{ls} = \arg \min_n \|g - An\|^2,$$  \hspace{1cm} (9)$$

where $\|\cdot\|$ is the Euclidean norm. This quadratic objective function has a unique solution, provided that $A$ is of full rank. In this study, we used the conjugate gradient algorithm [11] to perform the optimization.

### 3. PRELIMINARY RESULTS

#### 3.1 Evaluation Image Data

To demonstrate the proposed CAMM-based reconstruction approach, we used the 4D gated mathematical cardiac-torso (gMCAT) D1.01 phantom [12], which is a time sequence of 16 3D images. The field of view was 36 cm; the pixel size was 5.625mm. Poisson noise, at a level of 4 million total counts per 3D time-frame image, was introduced into the projections to simulate a clinical $\theta_{\text{deg}}$ study. No attenuation correction was used.

#### 3.2 Volumetric Mesh Generation

The key to the proposed approach lies in how to construct a CAMM that is compact and accurate for representing the volumetric image to be reconstructed. For this purpose we extended our method in [13] to the 3D case. This method consists of the following three steps: 1) extract a feature map $\sigma(x)$ from the image $f(x)$ based on the largest magnitude of its second directional derivatives; 2) apply the well-known Floyd-Steinberg error-diffusion algorithm, a method originally designed for
digital halftoning [14], to distribute mesh nodes non-uniformly in the 3D image domain, with density proportional to the feature map \( \sigma(p) \); and 3) use a 3D Delaunay triangulation algorithm [15] to connect the mesh nodes. The resulting mesh consists of tetrahedral elements that are automatically adapted to the content of the image.

To demonstrate the accuracy of the CAMM produced by this algorithm, we show in Fig. 1 some results obtained for a 3D frame from the phantom. Shown in Fig. 1(a) is a short-axis view of four slices selected in the vicinity of the heart from this original volumetric frame. Shown in Fig. 1(b) are these same four slices from a mesh representation of this volume obtained from our algorithm, in which 10,688 mesh nodes were used (only about 4% of the total number of voxels used in (a)). To quantify the accuracy of this mesh representation, we computed its peak-signal-to-noise ratio (PSNR) to be 42.8 dB. The PSNR is defined as

\[
PSNR = 10 \log \left( \frac{M \times N \times L \cdot f_{\text{max}}^2}{\|f - \hat{f}\|^2} \right) \text{dB}, \quad (6)
\]

where \( f \) and \( \hat{f} \) denote the original image and its mesh representation, respectively, \( f_{\text{max}} \) is the image peak value, and \( M \times N \times L \) is the image dimension.

These results demonstrate that the proposed mesh model can indeed provide an accurate representation of the volumetric image with a very small number of mesh nodes.

Of course, for tomographic image reconstruction the mesh structure has to be estimated from the observed data. The following procedure was demonstrated to work well in our previous studies [7]. First, the projection data are summed over the 16 gated frames. From these summed projections an image is reconstructed using the filtered back projection algorithm. The resulting image, denoted by \( \hat{f}(x) \), provides a rough estimate of the heart summed over all 16 frames. The mesh structure is then created based on \( \hat{f}(x) \) using the steps described above.

In Fig. 2 we show the mesh structure for a 2D slice obtained from the projection data. The mesh nodes are highlighted in Fig. 2 using bright dots. It is evident that the obtained mesh structure is well adapted to the content of the image, where mesh nodes have been automatically placed densely in the important heart regions, and sparsely in the background.

For better visualization purposes, additional results and animations are provided for the obtained 3D mesh structures at the following web site: http://www.ipl.iit.edu/brankov/Rotate.htm. These results demonstrate that the mesh structure produced by the proposed method is well adapted to the content of the 3D volumetric images.

3.3 Fully 3D CAMM Reconstruction

For validating the concept of the proposed CAMM based reconstruction methods, our initial work has been focused on using 2D models. These preliminary results indicate that the proposed method can outperform several existing pixel-based methods. We are currently applying the volumetric mesh model described above to fully 3D reconstruction.

Given the success we had using the 2D model, we expect that the use of a fully 3D CAMM could offer even...
greater advantage for image reconstruction. This is because a 3D CAMM can further exploit the redundancy among the different 2D slices in a volumetric frame, offering a much more compact representation than in the 2D case. This point is clearly demonstrated by the results in Fig. 1, where the number of mesh nodes used was only 4% of the number of voxels in the original volumetric frame, yet the mesh representation achieves a mean-square-error as low as 5.25×10−4 (assuming image dynamic range between 0 and 1). It is expected that such a great reduction in the number of unknowns by the mesh model would eventually lead to a very efficient reconstruction algorithm. We plan to furnish detailed, complete results of this fully 3D reconstruction method, along with comparisons to other methods [e.g., 16-18], by the time of the conference.

5. REFERENCES


